Investigation on the stability of the Vector Jiles-Atherton Model

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The paper presents a numerical procedure to discretize the differential equation arising in the vector Jiles-Atherton hysteresis model. In addition to the numerical procedure, the effect of parameters on the stability of the solution is also presented. In essence, the parameter identification through optimization have been discussed in detail that literally covers the Jiles-Atherton models response to the uni-directional alternating magnetic flux density input. The non-physical $b - h$ loop and convergence problems are faced when using vector Jiles-Atherton model in the magnetic field simulations. To account for the convergence issues, an adaptive time step control is implemented that fosters the simulation and enhances the convergence.

***Index Terms*—Differential Susceptibility, Magnetic Hysteresis, Optimization.**

I. Introduction

The phenomenological Jiles-Atherton (J-A) hysteresis model is one of the most extensively studied and used hysteresis model beside the mathematical Preisach model [1]. The J-A model possess some advantages over the Preisach model in terms of simplicity and computational robustness [2], [3]. In addition, the recent studies shows that the J-A model is well adapted in applications involving 2-D magnetic field problems [4]. Nevertheless, the latent complexities involved in the parameter identification and convergence issues in 2-D electromagnetic field problems merely justify the simplicity of the phenomenological hysteresis model. The numerical procedure used in the discretization of the vector J-A model differential equation can immediately reveal the sensitivity of the J-A model in response to the variation of the parameters. In addition, the convergence issue within the material model is severe enough to hinder its successful application in the numerical analysis of electromagnetic field. This paper aims to outline all the possible glitch in the vector J-A model, and address such problems with relevant rectification where necessary.

II. Methodology

A. Vector Jiles-Atherton Model

Bergqvist [5] in 1996 introduced the vector formulation of the original scalar J-A model. Taking the magnetic flux density $\boldsymbol{b}$ as the input variable, the change in magnetization $\boldsymbol{m}$ is given by the differential equation [6]:

(i) If $\boldsymbol{\chi}_{\mathrm{f}} \cdot \mathrm{d}\boldsymbol{h}_{\mathrm{e}} > 0$

$$\begin{aligned} \mathrm{d}\boldsymbol{m} = \boldsymbol{\nu}_0 \Bigg[\underline{\underline{\mathrm{I}}} + \frac{\boldsymbol{\chi}_{\mathrm{f}}}{|\boldsymbol{\chi}_{\mathrm{f}}|} \cdot \boldsymbol{\chi}_{\mathrm{f}} \cdot \left(\underline{\underline{\mathrm{I}}} - \underline{\underline{\alpha}} \right) + \underline{\underline{c}} \cdot \underline{\underline{\xi}} \\ \cdot \left(\underline{\underline{\mathrm{I}}} - \underline{\underline{\alpha}} \right) \Bigg]^{-1} \cdot \left[\frac{\boldsymbol{\chi}_{\mathrm{f}}}{|\boldsymbol{\chi}_{\mathrm{f}}|} \cdot \boldsymbol{\chi}_{\mathrm{f}} + \underline{\underline{c}} \cdot \underline{\underline{\xi}} \right] \cdot \mathrm{d}\boldsymbol{b}, \end{aligned} \tag{1}$$

(ii) If $\boldsymbol{\chi}_{\mathrm{f}} \cdot \mathrm{d}\boldsymbol{h}_{\mathrm{e}} \leq 0$

$$\mathrm{d}\boldsymbol{m} = \boldsymbol{\nu}_0 \left[\underline{\underline{\mathrm{I}}} + \underline{\underline{c}} \cdot \underline{\underline{\xi}} \cdot \left(\underline{\underline{\mathrm{I}}} - \underline{\underline{\alpha}} \right) \right]^{-1} \cdot \left[\underline{\underline{c}} \cdot \underline{\underline{\xi}} \right] \cdot \mathrm{d}\boldsymbol{b}, \tag{2}$$

where $\boldsymbol{\chi}_{\mathrm{f}} = \underline{\underline{k}}^{-1} \cdot (\boldsymbol{m}_{\mathrm{an}} - \boldsymbol{m})$ is an auxiliary vector quantity, tensors $\underline{\underline{k}}$, $\underline{\underline{\alpha}}$, and $\underline{\underline{c}}$ are the model parameters, $\underline{\underline{\xi}}$ is the differential anhysteretic susceptibility, $\underline{\underline{\mathrm{I}}}$ represents the unit tensor, ν_0 is the vacuum reluctivity, and $\boldsymbol{m}_{\mathrm{an}}$ represents the anhysteretic magnetization vector. The anhysteretic magnetization vector is a function of the effective field, $\boldsymbol{h}_{\mathrm{e}} = \boldsymbol{h} + \underline{\underline{\alpha}} \cdot \boldsymbol{m}$, and is collinear with it [6]:

$$\boldsymbol{m}_{\mathrm{an}} = m_{\mathrm{anx}}\mathbf{i} + m_{\mathrm{any}}\mathbf{j} = m_{\mathrm{an}}\left(|\boldsymbol{h}_{\mathrm{e}}|\right) \frac{\boldsymbol{h}_{\mathrm{e}}}{|\boldsymbol{h}_{\mathrm{e}}|}, \tag{3}$$

$$\underline{\underline{\xi}} = \begin{bmatrix} \dfrac{\partial m_{\mathrm{anx}}}{\partial h_{\mathrm{ex}}} & \dfrac{\partial m_{\mathrm{anx}}}{\partial h_{\mathrm{ey}}} \\ \dfrac{\partial m_{\mathrm{any}}}{\partial h_{\mathrm{ex}}} & \dfrac{\partial m_{\mathrm{any}}}{\partial h_{\mathrm{ey}}} \end{bmatrix}. \tag{4}$$

The classical Langevin and Brillouin functions have been commonly used to represent the anhysteretic magnetization curve given by (3), however various other sigmoid functions have been equally suitable in representing the lossless curve [7], [8]. The expressions for the elements of (4) are referred to [6]. Using (1)-(2) the differential reluctivity can be expressed as:

$$\frac{\mathrm{d}\boldsymbol{h}}{\mathrm{d}\boldsymbol{b}} = \nu_0 \underline{\underline{\mathrm{I}}} - \frac{\mathrm{d}\boldsymbol{m}}{\mathrm{d}\boldsymbol{b}}. \tag{5}$$

Equation (5) can be re-written as:

$$\frac{\mathrm{d}\boldsymbol{h}}{\mathrm{d}\boldsymbol{b}} = \underline{\underline{f}}\left(\boldsymbol{h}_{\mathrm{n+1}}, \boldsymbol{h}_{\mathrm{n}}, \boldsymbol{b}_{\mathrm{n+1}}, \boldsymbol{h}_{\mathrm{n}}\right), \tag{6}$$

where n and n + 1 denote the current and next time step values.

The discretization of (4) can be done using implicit midpoint method, backward Euler method or other linear multi-step methods. Often, the fourth order Runge-Kutta method have been used in solving (1)-(2). The underlying assumption that permits the use of the explicit Runge-Kutta method in solving (4) is to consider no irreversible effect taking place in the initial magnetization curve, especially near the origin. In essence, (2) follows at the startup of the simulation (i.e., $\mathrm{d}\boldsymbol{h} \geq 0$). Albeit, the condition $(\boldsymbol{m}_{\mathrm{an}} - \boldsymbol{m}_{\mathrm{irr}}) \cdot \mathrm{d}\boldsymbol{h}_{\mathrm{e}}$ in the inverse model does not allow the use of explicit numerical discretization scheme. The presence of parameter $\underline{\underline{\alpha}}$ in $\boldsymbol{h}_{\mathrm{e}} = \boldsymbol{h} + \underline{\underline{\alpha}} \cdot \boldsymbol{m}$ makes the hysteresis model implicit [5]. Equation (6) is solved using (1)-(2), and hence the next time step value of the magnetic field can be computed as:

$$\boldsymbol{h}_{\mathrm{n+1}} = \boldsymbol{h}_{\mathrm{n}} + \frac{\mathrm{d}\boldsymbol{h}}{\mathrm{d}\boldsymbol{b}} \cdot (\boldsymbol{b}_{\mathrm{n+1}} - \boldsymbol{b}_{\mathrm{n}}) \,. \tag{7}$$

B. Parameter Identification and Essential criteria

Several optimization schemes are used to obtain the J-A model parameters. Derivative free methods like genetic algorithm, particle-swarm method, simulated annealing, pattern search method, and others have become very popular in optimizing the J-A model parameters [9],[10]. During parameter optimization, the cost function is evaluated at randomly varied parameter values. At certain combination of the parameters, the simulation result would give non-physical $b-h$ loop. The negative value of the differential susceptibility is non-physical for a ferromagnetic material (see Fig. 1). Hence, in-order to avoid non-physical $b-h$ characteristic, the following essential criteria should be included in the optimization algorithm [11]:

(i) $\dfrac{M_{\mathrm{s}}\alpha}{3a} < 1$,

(ii) $0 < c < 1$ and $k > 0$.

C. Convergence Problem

Although, the implicit method is preferred, the non-linear iterations do not converge at all. In essence, a small $\mathrm{d}\boldsymbol{b}$ value must be used, however, to speed up the simulation time the authors prefer to use an adaptive time step control in the algorithm.

III. Results and Discussion

The simulation result shown in Fig. 1 and Fig. 2 clearly emphasize the importance of essential criteria. Both the results in Fig. 1 and Fig. 2 suffer from the convergence problem due to the large and fixed $\mathrm{d}\boldsymbol{b}$. In the full paper, the result of using an adaptive time-step control and the effect of such adaptive control in solving 2-D magnetic field problem using finite element method will be presented.

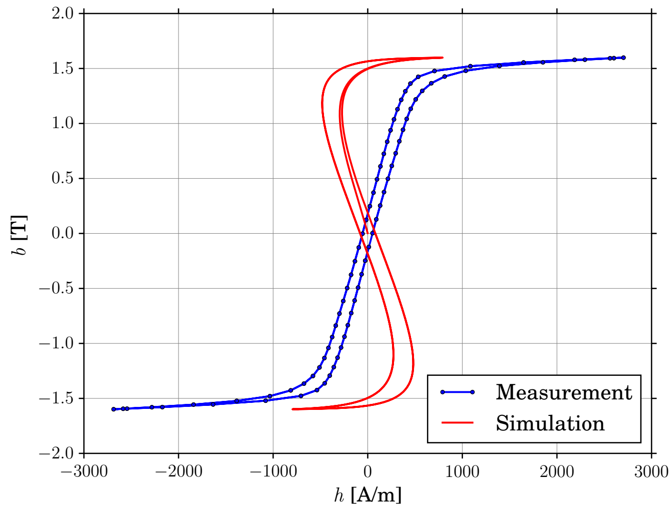

Fig. 1. Essential criteria violated.

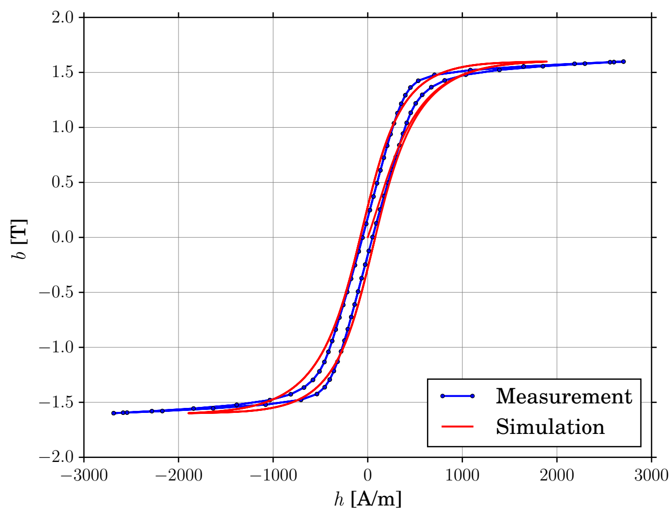

Fig. 2. Essential criteria fulfilled.

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